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Turbulence modelling for unsteady separated flows -

abstract for NASA/AFOSR/ARO Workshop on Unsteady Separation

Peter Bradshaw, Feb. 1990

1. Unsteadiness

The exact transport equations for turbulent (Reynolds) stresses have left-hand sides representing the "substantial derivatives" of the Reynolds stresses, i.e. the rates of change of stress with respect to time, as seen by an observer following the mean motion of the fluid. Here the "mean" is a statistical average for the turbulent motion, distinguished from the ordered unsteadiness on which it is superimposed: for a turbomachine blade or a cyclically-pitching airfoil, the mean is a phase average (Fig. 1: see Ref. 1 for a practical discussion). Written in coordinates fixed with respect to a solid surface, the substantial derivative appears partly as an Eulerian time derivative at given spatial coordinate position and partly as a spatial derivative.

If the Reynolds-stress transport equations are modelled term by term ("stress-transport" or "second-order" models), the left-hand sides are left in exact form. The right-hand sides of the exact equations contain no time derivatives and there is no justification for introducing them in a model. Therefore the applicability of a stress-transport model to unsteady flow can be judged on its performance in steady flow: a model that behaves well in steady flows with rapid streamwise changes in stress (implying a large substantial derivative on the left-hand side) will behave equally well in unsteady flows where the left-hand side is equally large because of rapid timewise and/or streamwise changes.

This conclusion is true only of stress-transport models: models which ignore or approximate the left-hand sides cannot be judged in this way, but are necessarily suspect in any flow where the left-hand side is large. It seems inescapable that the only candidates for rapidly-changing unsteady flows are stress-transport models (e.g. Refs. 2, 3). Any model based on eddy viscosity relates the turbulent stresses to the local mean velocity gradients, which amounts to ignoring the left-hand sides of the Reynolds-stress transport equations. (This is true even for two-equation models, which use transport equations for turbulent energy and dissipation

rate.) Algebraic stress models are based on an approximation to the left-hand sides which can easily be shown to be poor in rapidly-changing flows.

Clearly, unsteadiness can lead to secondary effects (e.g. appearance of concentrated spanwise vortices in a boundary layer or vortex street) which would defeat a turbulence model even in steady flow, so that passing the "left-hand-side" test is necessary but not sufficient.

2. Separation

Separation presents two specific problems to a turbulence model:--

(i) Prediction of the flow near separation depends critically on the "near-wall" part of the turbulence model. Several workers are currently studying this problem (Refs. 4-9), but all are using conventional models for the correlations between the pressure fluctuation and the velocity-gradient fluctuations. These correlations redistribute contributions to the Reynolds-stress tensor among the different components, and their modelling is a key part of any transport-equation method. Current practice is to relate the "redistribution" terms to local turbulence quantities and mean-flow gradients, but this is essentially risky because the pressure fluctuation at a point depends on an integral of the velocity fluctuations over a nominally infinite volume. Comparison with turbulence simulation data (Ref. 10) show that this "local" assumption breaks down very badly in the viscous wall region, where turbulence quantities and mean-flow gradients are changing rapidly with distance from the surface. The models can always be forced to reproduce the "law of the wall" in attached flows, simply by making the empirical coefficients functions of a Reynolds number related to the dimensionless wall distance y^+ : however the flaw in the basic assumptions suggests that the models will break down near separation where the law of the wall no longer holds.

(ii) Downstream of separation, a boundary layer changes gradually to a mixing layer. Even in the simplest case of formation of a mixing layer from the boundary layer at exit from a jet nozzle, the effects of initial conditions persist for extremely long distances downstream. If the turbulence model does not predict boundary layers and (asymptotic) mixing layers adequately with the same set of coefficients, the coefficients must be interpolated in the streamwise direction. This is the "zonal modelling" technique (Ref. 11): it is also applicable in ad hoc corrections of the defects of turbulence models in special zones like imbedded vortices or shock-wave interactions.

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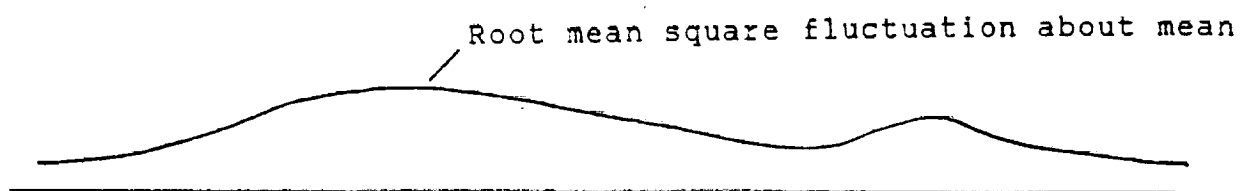
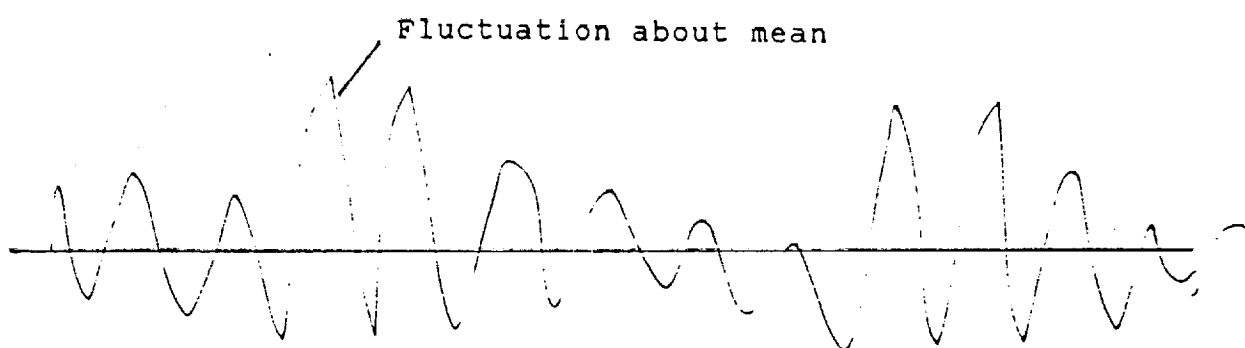
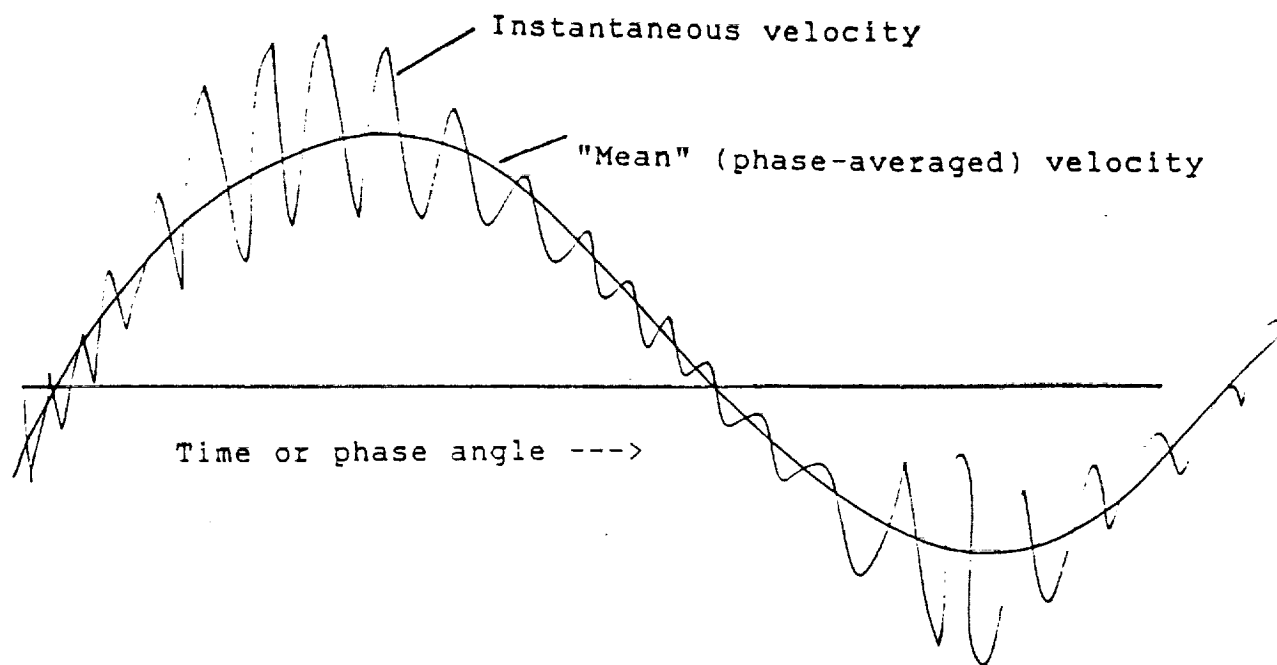


Fig. 1 Phase Averaging in Cyclically-Unsteady Flow

NASA/AFOSR/ARO Workshop on Unsteady Separation

Turbulence modelling for unsteady separated flows

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1. Unsteadiness

1.1 Transport equations – general

- Exact “transport” equations govern turbulent (Reynolds) stresses.
- As for all transport equations, their left-hand sides represent the mean rates of change of the transported quantity with time, as seen by an observer following the mean motion of the fluid.
- Here the “mean” is a statistical average for the turbulent motion, distinguished from the ordered unsteadiness on which it is superimposed.

1.2 Definitions of averages

- For a turbomachine blade or a cyclically-pitching airfoil, we usually choose the mean to be a phase average.
- In coordinates fixed with respect to a solid surface, the mean rate of change seen by a moving observer appears as:
a time derivative at given spatial position
plus a spatial derivative (which varies with time).

1.3 Transport equations for Reynolds stresses

- In “stress-transport” or “second-order” models, the Reynolds-stress transport equations are modelled term by term.
- The rate of change of Reynolds stress on the left-hand side of each equation is left in exact form – no modelling.
- The right-hand sides of the exact equations contain no time derivatives ...

...so there is no justification for introducing time derivatives in a model of the right-hand sides!

- Because the left-hand side depends only on the sum of the time derivatives and space derivatives, the applicability of a stress-transport model to unsteady flow can be judged on its performance in steady flow...

1.4 Transport equations for Reynolds stresses – continued

- Suppose we have a model that behaves well in steady flows with rapid streamwise changes in stress (implying a large “left-hand side”)...
- ...it should behave equally well in unsteady flows where the left-hand side is equally large, because of rapid time-wise (and/or streamwise) changes.
- This conclusion is true only of stress-transport models.
- The unsteady-flow performance of models which ignore or approximate the left-hand sides of the Reynolds-stress transport equations cannot be estimated in this way.
- Such models are necessarily suspect in any flow where the left-hand sides (rates of change of Reynolds stresses) are large.

1.5 Eddy-viscosity models

- Any model based on eddy viscosity relates the turbulent stresses to the local mean velocity gradients, which amounts to ignoring the left-hand sides of the Reynolds-stress transport equations.

- This is true even for “two-equation” models such as the k, ϵ model.

These models use transport equations for turbulent energy and dissipation rate but not for the individual stresses.

- Algebraic stress models were intended to improve on the assumption of isotropic eddy viscosity in the “two-equation” models.

However they are based on an approximation to the left-hand sides which is poor in rapidly-changing flows.

1.6 Turbulence models for unsteady flows – conclusions

- It seems inescapable that the only plausible candidates for rapidly-changing unsteady flows are stress-transport models.
- Unfortunately, stress-transport models are harder to program, and taken longer to run, than eddy-viscosity models. The more independent variables you have, the bigger the computational disadvantage.
- As H.L. Mencken said *“To every difficult question there is a simple answer – which is wrong”*.
- Clearly, unsteadiness can lead to new secondary effects, e.g. the appearance of concentrated spanwise vortices in a boundary layer or vortex street.
- These might defeat a turbulence model even in steady flow, so that passing the “left-hand-side” test is necessary but not sufficient.

2. Separation

- Separation presents two specific problems to a turbulence model:–
 - (i) Prediction of the flow near separation depends critically on the “near-wall” part of the turbulence model.
 - (ii) Downstream of separation, a boundary layer changes gradually to a mixing layer.

2.1 Near-wall modelling

- Several workers are currently studying this problem, using stress-transport equation models. Here, the difficulty is to model transport *normal* to the surface.
- All are using conventional models for the correlations between the pressure fluctuation and the velocity-gradient fluctuations. (These correlations redistribute contributions to the Reynolds-stress tensor among the different components, and their modelling is a key part of any transport-equation method.)
- Current practice is to relate the “redistribution” terms to local turbulence quantities and mean-flow gradients.

2.2 Near-wall modelling – continued

- “Local” modelling is always risky, because the pressure fluctuation at a point depends on an integral of the velocity fluctuations over a nominally infinite volume.
- Comparisons with turbulence simulation data show that this “local” assumption breaks down very badly in the viscous wall region...
...where turbulence quantities and mean-flow gradients are changing rapidly with distance from the surface.
- Any model can be forced to reproduce the “law of the wall” in attached flows, simply by making the empirical coefficients functions of a Reynolds number related to the dimensionless wall distance y^+ .
- However the flaw in the basic assumptions suggests that these attached-flow models will break down near, and after, separation where the law of the wall no longer holds.

2.3 Modelling of separated shear layers

- Even in the simplest case of formation of a mixing layer from the boundary layer at exit from a jet nozzle, the effects of initial conditions persist for extremely long distances downstream.
- If the turbulence model does not predict boundary layers and (asymptotic) mixing layers adequately with the same set of coefficients, the coefficients must be interpolated in the streamwise direction.
- This is the “zonal modelling” technique: it is also applicable in *ad hoc* corrections of the defects of turbulence models in special zones like imbedded vortices or shock-wave interactions.
- Zonal modelling – model coefficients depending on local flow type – is the only logical alternative to the dream of a “universal” turbulence model.

Conclusions

- There is a universal turbulence model – the 3D, time-dependent Navier-Stokes equations. Any simpler model is likely to be less accurate or less widely applicable!
- Turbulence modelling is caught between the Navier-Stokes equations and Mencken's Theorem.
- It will be a long time before numerically-exact Navier-Stokes simulations can produce useful results for unsteady flows in engineering geometries...
- ...and until then we must use the “best-buy” models.
- A major conclusion of this review is that only Reynolds-stress transport models, using PDE “transport” equations for the Reynolds stresses, can incorporate the physics of unsteady flows.